



AFRL-OSR-VA-TR-2013-0291

(DARPA) EXPLOITING MANY-BODY BUS STATES FOR MULTI-
QUBIT ENTANGLEMENT

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06/06/2013
Final Report

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REPORT DOCUMENTATION PAGE					Form Approved OMB No. 0704-0188	
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1. REPORT DATE (DD-MM-YYYY)		2. REPORT TYPE			3. DATES COVERED (From - To)	
4. TITLE AND SUBTITLE				5a. CONTRACT NUMBER		
				5b. GRANT NUMBER		
				5c. PROGRAM ELEMENT NUMBER		
6. AUTHOR(S)				5d. PROJECT NUMBER		
				5e. TASK NUMBER		
				5f. WORK UNIT NUMBER		
7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES)					8. PERFORMING ORGANIZATION REPORT NUMBER	
9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES)					10. SPONSOR/MONITOR'S ACRONYM(S)	
					11. SPONSOR/MONITOR'S REPORT NUMBER(S)	
12. DISTRIBUTION/AVAILABILITY STATEMENT						
13. SUPPLEMENTARY NOTES						
14. ABSTRACT						
15. SUBJECT TERMS						
16. SECURITY CLASSIFICATION OF:			17. LIMITATION OF ABSTRACT	18. NUMBER OF PAGES	19a. NAME OF RESPONSIBLE PERSON	
a. REPORT	b. ABSTRACT	c. THIS PAGE			19b. TELEPHONE NUMBER (Include area code)	

AFOSR Final Report: FA9550-09-1-0040 - (DARPA) “Exploiting many-body bus states for multi-qubit entanglement,” Mark Friesen, University of Wisconsin-Madison (PI) and Xuedong Hu (State University of New York at Buffalo).

Reporting period: December 1, 2008 to January 31, 2013.

Summary

A spin bus is formed from a linear array of quantum dots, with one electron per dot, and strong, always-on exchange interactions. When the spin bus is coupled to external spin qubits, it provides a mechanism for mediating effective long-range interactions.

In this project, our efforts were divided between three main tasks: (1) to develop multi-qubit entangling protocols using the spin-bus; (2) to clarify bus decoherence properties and to optimize the robustness of bus operations; (3) to investigate methods of bus operation that could be enhanced in the vicinity of a quantum phase transition.

Our main results are summarized as follows. We contrasted the characters of even and odd-size chains when they are coupled to external qubits. We investigated how the strain-induced nuclear quadrupole interaction influences the degree of nuclear spin polarization driven by optical pumping in self-assembled quantum dots. We proposed a method for controlling anisotropic interactions between spins arranged in a bus geometry. We studied electron-phonon interaction induced decoherence between two-electron singlet and triplet states in a semiconductor double quantum dot using a spin-boson model. We studied quantum-state transfer through a strongly coupled antiferromagnetic spin chain. We studied how the fidelity of a spin-1/2 Heisenberg chain as a spin bus could be affected by static random exchange couplings and magnetic fields. We developed a class of “mediated” gates for spin qubits, which act on nonproximal spins via intermediate ancilla qubits. We studied electron-spin-photon coupling in a single-spin double quantum dot embedded in a superconducting stripline cavity. We studied controllable exchange coupling between two singlet-triplet qubits. We derived a generalized form of the electric dipole spin resonance Hamiltonian in the presence of the spin-orbit interaction for single spins in an elliptic quantum dot subject to an arbitrary applied magnetic field. We devised a platform for noise-resistant quantum computing using the valley degree of freedom of Si quantum dots. We investigated the nonadiabatic implementation of an adiabatic quantum teleportation protocol, finding that perfect fidelity can be achieved through resonance. We showed that a double quantum dot coupled locally to a spin chain provides an alternative and efficient probe of a quantum phase transition. We showed that, if the rotation axes can be tuned arbitrarily in a fixed plane, then two rotation steps are sufficient for implementing a single-qubit gate, and one rotation step is sufficient for implementing a state transformation. We investigated the hyperfine-induced dephasing of two-electron-spin states in a double quantum dot with a finite singlet-triplet splitting.

Report

Here, we describe our main results, either published or submitted for publication.

A strongly coupled Heisenberg chain provides an important channel for quantum communication through its many-body ground state. Yet, the nature of the effective interactions and the ability to mediate long-range entanglement differs significantly for chains of opposite parity. In Ref. [1], we contrast the characters of even and odd-size chains when they are coupled to external qubits. Additional parity effects emerge in both cases, depending on the positions of the attached qubits. Some striking results include (i) the emergence of maximal entanglement and (ii) spin-chain Ruderman-Kittel-Kasuya-Yosida interactions for qubits attached to an even chain, and (iii) the ability of chains of either parity to mediate qubit entanglement that is undiminished by distance.

In Ref. [2], we study the suppression of spin decoherence via nuclear spin polarization. In particular, we investigate how the strain-induced nuclear quadrupole interaction influences the degree of nuclear spin polarization driven by optical pumping in self-assembled quantum dots. Our calculation shows that the achievable nuclear spin polarization in $\text{In}_x\text{Ga}_{1-x}\text{As}$ quantum dots is related to the concentration of indium and the resulting strain distribution in the dots. The interplay between the nuclear quadrupole interaction and Zeeman splitting leads to interesting features in the magnetic field dependence of the nuclear spin polarization. Our results are in qualitative agreement with measured nuclear spin polarization by various experimental groups.

The exchange coupling between quantum dot spin qubits is isotropic, which restricts the types of quantum gates that can be formed. In Ref. [3], we propose a method for controlling anisotropic interactions between spins arranged in a bus geometry. The symmetry is broken by an external magnetic field, resulting in XXZ -type interactions that can efficiently generate maximally entangled Greenberger-Horne-Zeilinger states or universal gate sets for exchange-only quantum computing. We exploit the XXZ couplings to propose a qubit scheme, based on double dots.

In Ref. [4], we study electron-phonon interaction induced decoherence between two-electron singlet and triplet states in a semiconductor double quantum dot using a spin-boson model. We investigate the onset and time evolution of this dephasing, and study its dependence on quantum dot parameters such as dot size and double dot separations, as well as the host materials (GaAs and Si). At the short time limit, electron-phonon interaction only causes an incomplete initial Gaussian decay of the off-diagonal density matrix element in the singlet-triplet Hilbert space; a complete long-time exponential decay due to phonon relaxation would eventually dominate over two-spin decoherence. We analyze two-spin decoherence in both symmetric and biased double quantum dots, identifying their difference in electron-phonon coupling and the relevant consequences.

In Ref. [5], we study quantum-state transfer (QST) through a strongly coupled antiferromagnetic spin chain (acting as a spin bus), between weakly coupled external qubits. By treating the weak coupling as a perturbation, we find that QST is enabled specifically by the second-order terms in the perturbative expansion. We show that QST is robust against disorder in the couplings, either within the bus or to the external qubits. We find that the protocol works when the qubits are attached to any node on an even-size bus or to the antiferromagnetic nodes on an odd-size bus. The optimal time for QST is found to depend nonmonotonically on qubit separation.

A strongly coupled spin chain can mediate long-distance effective couplings or entanglement between remote qubits and can be used as a quantum data bus. In Ref. [6], we study how the fidelity of a spin-1/2 Heisenberg chain as a spin bus is affected by static random exchange couplings and magnetic fields. We find that, while nonuniform exchange couplings preserve the isotropy of the qubit effective couplings, they cause the energy levels, the eigenstates, and the magnitude of the couplings to vary locally. On the other hand, random local magnetic fields lead to an avoided level crossing for the bus ground-state manifold and cause the effective qubit couplings to be anisotropic. Interestingly, the total magnetic moment of the ground state of an odd-size bus may not be parallel to the average magnetic field. Its alignment depends on both the direction of the average field and the field distribution, in contrast with the ground state of a single spin which always aligns with the applied magnetic field to minimize the Zeeman energy. Lastly, we calculate sensitivities of the spin bus to such local variations, which are potentially useful for evaluating decoherence when dynamical fluctuations in the exchange coupling or magnetic field are considered.

In a typical quantum circuit, nonlocal quantum gates are applied to nonproximal qubits. If the underlying physical interactions are short-range (e.g., exchange interactions between spins), intermediate SWAP operations must be introduced, thus increasing the circuit depth. In Ref. [7], we develop a class of “mediated” gates for spin qubits, which act on nonproximal spins via intermediate ancilla qubits. At the end of the operation, the ancillae return to their initial states. We show how these mediated gates can be used (1) to generate arbitrary quantum states and (2) to construct arbitrary quantum gates. We provide some explicit examples of circuits that generate common states [e.g., Bell, Greenberger-Horne-Zeilinger (GHZ), W , and cluster states] and gates (e.g., $\sqrt{\text{SWAP}}$, SWAP, CNOT, and Toffoli gates). We show that the depths of these circuits are often shorter than those of conventional SWAP-based circuits. We also provide an explicit experimental proposal for implementing a mediated gate in a triple-quantum-dot system.

We have explored other approaches to achieve on-chip quantum communication beyond spin buses. In particular, in Ref. [8] we study electron-spin-photon coupling in a single-spin double quantum dot embedded in a superconducting stripline cavity. With an external magnetic field, we show that either a spin-orbit interaction (for InAs) or an inhomogeneous magnetic field (for Si and GaAs)

could produce a strong spin-photon coupling, with a coupling strength of the order of 1 MHz. With an isotopically purified Si double dot, which has a very long spin coherence time for the electron, it is possible to reach the strong-coupling limit between the spin and the cavity photon, as in cavity quantum electrodynamics. The coupling strength and relaxation rates are calculated based on parameters of existing devices, making this proposal experimentally feasible.

We have explored non-Heisenberg couplings between spin-bus nodes. In Ref. [9], we study controllable exchange coupling between two singlet-triplet qubits. We start from the original second quantized Hamiltonian of a quadruple quantum dot system and obtain the effective spin-spin interaction between the two qubits using the projection operator method. Under a strong uniform external magnetic field and an inhomogeneous local micromagnetic field, the effective interqubit coupling is of the Ising type, and the coupling strength can be expressed in terms of quantum dot parameters. Finally, we discuss how to generate various two-qubit operations using this controllable coupling, such as entanglement generation, and a controlled-NOT gate.

In Ref. [10], we explore the effect of spin-orbit interaction in coupled quantum dots. Specifically, we derive a generalized form of the electric dipole spin resonance (EDSR) Hamiltonian in the presence of the spin-orbit interaction for single spins in an elliptic quantum dot (QD) subject to an arbitrary (in both direction and magnitude) applied magnetic field. We predict a nonlinear behavior of the Rabi frequency as a function of the magnetic field for sufficiently large Zeeman energies, and present a microscopic expression for the anisotropic electron g tensor. Similarly, an EDSR Hamiltonian is devised for two spins confined in a double quantum dot (DQD), where coherent Rabi oscillations between the singlet and triplet states are induced by jittering the inter-dot distance at the resonance frequency. Finally, we calculate two-electron-spin relaxation rates due to phonon emission, for both in-plane and perpendicular magnetic fields. Our results have immediate applications to current EDSR experiments on nanowire QDs, g -factor optimization of confined carriers, and spin decay measurements in DQD spin-orbit qubits.

In Ref. [11], we devise a platform for noise-resistant quantum computing using the valley degree of freedom of Si quantum dots. The qubit is encoded in two polarized $(1,1)$ spin-triplet states with different valley compositions in a double quantum dot, with a Zeeman field enabling unambiguous initialization. A top gate gives a difference in the valley splitting between the dots, allowing controllable interdot tunneling between opposite valley eigenstates, which enables one-qubit rotations. Two-qubit operations rely on a stripline resonator, and readout on charge sensing. Sensitivity to charge and spin fluctuations is determined by intervalley processes and is greatly reduced as compared to conventional spin and charge qubits. We describe a valley echo for further noise suppression.

In Ref. [12], we investigate the nonadiabatic implementation of an adiabatic quantum teleportation protocol, finding that perfect fidelity can be achieved

through resonance. We clarify the physical mechanisms of teleportation, for three qubits, by mapping their dynamics onto two parallel and mutually coherent adiabatic passage channels. By transforming into the adiabatic frame, we explain the resonance by analogy with the magnetic resonance of a spin-1/2 particle. Our results establish a fast and robust method for transferring quantum states and suggest an alternative route toward high-precision quantum gates.

Quantum phase transitions (QPTs) in qubit systems are known to produce singularities in the entanglement, which could in turn be used to probe the QPT. Recent proposals have suggested that the QPT in a spin chain could be probed via the entanglement between external qubits coupled to the spin chain. Such experiments may be technically challenging, because the probe qubits are nonlocal. In Ref. [13], we show that a double quantum dot coupled locally to a spin chain provides an alternative and efficient probe of a QPT. To demonstrate this method in a simple geometry, we propose an experiment to observe a QPT in a triple quantum dot, based on the well-known singlet projection technique.

Any single-qubit unitary operation or quantum gate can be considered a rotation. Typical experimental implementations of single-qubit gates involve two or three fixed rotation axes, and up to three rotation steps. In Ref. [14], we show that, if the rotation axes can be tuned arbitrarily in a fixed plane, then two rotation steps are sufficient for implementing a single-qubit gate, and one rotation step is sufficient for implementing a state transformation. The results are relevant for “exchange-only” logical qubits encoded in three-spin blocks, which are important for universal quantum computation in decoherence free subsystems and subspaces.

In Ref. [15], we investigate the hyperfine-induced dephasing of two-electron-spin states in a double quantum dot with a finite singlet-triplet splitting J . In particular, we derive an effective pure dephasing Hamiltonian, which is valid when the hyperfine-induced mixing is suppressed due to the relatively large J and the external magnetic field. Using both a quantum theory based on resummation of ring diagrams and semiclassical methods, we identify the dominant dephasing processes in regimes defined by values of the external magnetic field, the singlet-triplet splitting, and inhomogeneity in the total effective magnetic field. We address both free induction and Hahn echo decay of superposition of singlet and unpolarized triplet states (both cases are relevant for singlet-triplet qubits realized in double quantum dots). We also study hyperfine-induced exchange gate errors for two single-spin qubits. Results for III-V semiconductors as well as silicon-based quantum dots are presented.

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